1. Let $\boldsymbol{A} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

(a) Find

(i) $\boldsymbol{A}^{-1}$;

(ii) $\boldsymbol{A}^2$.

(4)

Let $\boldsymbol{B} = \begin{pmatrix} p & 2 \\ 0 & q \end{pmatrix}$.

(b) Given that $2\boldsymbol{A} + \boldsymbol{B} = \begin{pmatrix} 2 & 6 \\ 4 & 3 \end{pmatrix}$, find the value of p and of q.

(3)

(c) Hence find $\boldsymbol{A}^{-1}\boldsymbol{B}$.

(2)

(d) Let $\boldsymbol{X}$ be a 2×2 matrix such that $\boldsymbol{AX} = \boldsymbol{B}$. Find $\boldsymbol{X}$.

(2)

(Total 11 marks)

2. If $A = \begin{pmatrix} 2p & 3 \\ -4p & p \end{pmatrix}$ and det $A = 14$, find the possible values of p.

Working:

Answer:

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(Total 4 marks)

3. Let $M = \begin{pmatrix} a & 2 \\ 2 & -1 \end{pmatrix}$, where a $\in \mathbb{Z}$.

(a) Find M^2 in terms of a.

(4)

(b) If M^2 is equal to $\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}$, find the value of a.

(2)

(c) Using this value of a, find M^{-1} and **hence** solve the system of equations:

$$-x + 2y = -3$$
$$2x - y = 3$$

(6)

(Total 12 marks)

4. A and B are 2×2 matrices, where $A = \begin{bmatrix} 5 & 2 \\ 2 & 0 \end{bmatrix}$ and $BA = \begin{bmatrix} 11 & 2 \\ 44 & 8 \end{bmatrix}$. Find B.

Working:

Answer:

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(Total 4 marks)

5. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$.

Find

(a) $\boldsymbol{A} + \boldsymbol{B}$;

(2)

(b) $-3\boldsymbol{A}$;

(2)

(c) $\boldsymbol{AB}$.

(3)

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(Total 7 marks)

6. Let $\boldsymbol{A} = \begin{pmatrix} 3 & 2 \\ k & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find, in terms of k,

(a) $2\boldsymbol{A} - \boldsymbol{B}$;

(b) $\det(2\boldsymbol{A} - \boldsymbol{B})$.

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(Total 6 marks)

7. The function f is given by $f(x) = mx^3 + nx^2 + px + q$, where m, n, p, q are integers.
The graph of f passes through the point (0, 0).

(a) Write down the value of q.

(1)

The graph of f also passes through the point (3, 18).

(b) Show that $27m + 9n + 3p = 18$.

The graph of f also passes through the points (1, 0) and (–1, –10).

(2)

(c) Write down the other two linear equations in m, n and p.

(2)

(d) (i) Write down these three equations as a matrix equation.

(ii) Solve this matrix equation.

(6)

(e) The function f can also be written $f(x) = x(x-1)(rx-s)$ where r and s are integers. Find r and s.

(3)
(Total 14 marks)

8. Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix}$.

(a) Write down the inverse, $\boldsymbol{A}^{-1}$.

(2)

(b) $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{X}$ are also 2×2 matrices.

(i) Given that $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C}$, express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$, $\boldsymbol{B}$ and $\boldsymbol{C}$.

(ii) Given that $\boldsymbol{B} = \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix}$, and $\boldsymbol{C} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$, find $\boldsymbol{X}$.

(4)
(Total 6 marks)

9. (a) Write down the inverse of the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & -3 & 1 \\ 2 & 2 & -1 \\ 1 & -5 & 3 \end{pmatrix}$

(b) **Hence** solve the simultaneous equations

$x - 3y + z = 1$

$2x + 2y - z = 2$

$x - 5y + 3z = 3$

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(Total 6 marks)

10. Let S_n be the sum of the first n terms of the arithmetic series $2 + 4 + 6 + \ldots$.

(a) Find

(i) S_4;

(ii) S_{100}.

(4)

Let $\boldsymbol{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(b) (i) Find $\boldsymbol{M}^2$.

(ii) Show that $\boldsymbol{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$.

(5)

It may now be assumed that $\boldsymbol{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$, for $n \geq 4$. The sum $\boldsymbol{T}_n$ is defined by

$$\boldsymbol{T}_n = \boldsymbol{M}^1 + \boldsymbol{M}^2 + \boldsymbol{M}^3 + ... + \boldsymbol{M}^n .$$

(c) (i) Write down $\boldsymbol{M}^4$.

(ii) Find $\boldsymbol{T}_4$.

(4)

(d) Using your results from part (a) (ii), find $\boldsymbol{T}_{100}$.

(3)
(Total 16 marks)

11. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 18 \\ 23 \\ 13 \end{pmatrix}$ and $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) Write down the inverse matrix $\boldsymbol{A}^{-1}$.

(b) Consider the equation $\boldsymbol{AX} = \boldsymbol{B}$.

(i) Express $\boldsymbol{X}$ in terms of $\boldsymbol{A}^{-1}$ and $\boldsymbol{B}$.

(ii) **Hence**, solve for $\boldsymbol{X}$.

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(Total 6 marks)